

Assignment Music and Mathematics

Guitar Acoustics

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1 Abstract

This paper explains some topics on guitar acoustics, starting with some basics about the design and construction of guitars, to be able to fully understand how it works.

Although I made this work as a task for the course 'Music and Mathematics', the topics handled are more about physics, whereas mathematics provides us with nice ways to gain insight in the physical processes.

2 A Little Bit of History

Guitar-like instruments have existed since ancient times, but the first written mention of the guitar proper is from the 14th century. In its earliest form it had three double courses (pairs) of strings plus a single string (the highest). The modern six-string guitar is a descendant of the sixteenth-century Spanish vihuela, which has its roots in antiquity. The guitar became popular in other European countries in the 16th and 17th centuries, and by the late 17th century a fifth course of strings had been added below the other four.

Although Boccherini and other composers of the eighteenth century included the guitar in some of their chamber music, the establishment of the guitar as a concert instrument took place largely in the nineteenth century. Fernando Sor (1778-1839) was the first of a long line of Spanish virtuosos and composers for the guitar.

The Spanish luthier Antonio de Torres (1817-1892) contributed much to the development of the modern classical guitar when he enlarged the body and introduced a fan-shaped pattern of braces to the top plate. Francisco Tarrega (1852-1909), perhaps the greatest of all nineteenth century players, introduced the apoyando stroke and generally extended the expressive capabilities

of the guitar.

Guitars ranging from contrabass to treble, and with varying numbers of strings are played in Spain and Latin America. The twelve-string guitar has six double courses in standard tuning. The Hawaiian, or steel, guitar is laid across the knees of the player, who stops the metal strings by gliding a metal bar along the neck. The strings are usually tuned to the notes of a given chord.

The electric guitar, developed for popular music in the United States in the 1930s, usually has a solid body. The sound of its strings is both amplified and manipulated electronically by the performer. American musician and inventor Les Paul developed prototypes for the solid-bodied electric guitar and popularized the instrument beginning in the 1940s.

In the early 1940s, a California inventor, Leo Fender, made some custom guitars and amplifiers in his radio shop and already was working on an amplifier (with no controls) and a matching lap steel guitar (with tone and volume controls). This was typical of the way the electric guitar was viewed at this time, as a total package, and not as an individual instrument. With his knowledge of existing technologies, he knew he could improve on the amplified hollow-body instruments, and he did. In 1948 he developed the legendary *Telecaster* (originally named the *Broadcaster*). The *Tele*, as it became affectionately called, was the first solid body electric Spanish-style guitar ever to go into commercial production.

3 Design and Construction of Guitars

The modern guitar, as shown in figure 1, has 6 strings, about 65 cm in length, tuned to $E_2, A_2, D_3, G_3, B_3, E_4$. (with frequencies $f = 82, 110, 147, 196, 247$ and 330 Hz). The top is usually cut from spruce or redwood, planed to a thickness of about 2.5 mm. The back, also about 2.5 mm thick, is usually a hardwood, such as rosewood, mahogany, or maple. Both the top and back plates are braced, the bracing of the top plate being one of the critical design parameters.

Acoustic guitars generally fall into one of four families of design: classical, flamenco, flat top (also known as the folk guitar) and arch top. Classical and flamenco guitars have nylon strings; flat top and arch top guitars have steel strings. Steel string guitars usually have a steel rod embedded inside the neck, and their soundboards are provided with crossed bracing.

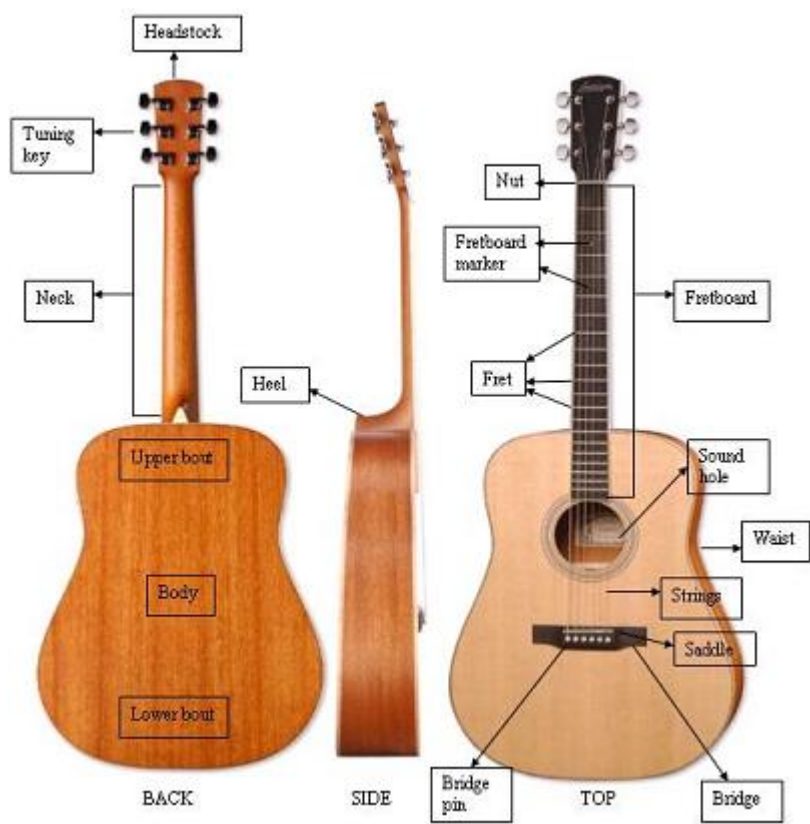


Figure 1: Guitar Anatomy

3.1 The Strings

The pitch of a vibrating string depends on four things:

- The **mass of the string**: more massive strings vibrate more slowly. On steel string guitars, the strings get thicker from high to low. On classical guitars, the size change is complicated by a change in density: the low density nylon strings get thicker from the E to B to G; then the higher density wire-wound nylon strings get thicker from D to A to E.
- The frequency can also be changed by changing **the tension in the string** using the tuning pegs: tighter gives higher pitch. This is what you do when you tune up.
- The frequency also depends on **the length of the string** that is free to vibrate. In playing, you change this by holding the string firmly against the fingerboard with a finger of the left hand. Shortening the string (stopping it on a higher fret) gives higher pitch.
- Finally there is the **mode of vibration**, which will be discussed later.

The strings themselves make hardly any noise: they are thin and slip easily through the air without making much of disturbance - and a sound wave is a disturbance of the air. An electric guitar played without an amplifier makes little noise, and an acoustic guitar would be much quieter without the vibrations of its bridge and body. In an acoustic guitar, the vibration of the string is transferred via the bridge and saddle to the top plate body of the guitar.

3.2 The Body

The body serves to transmit the vibration of the bridge into vibration of the air around it. For this it needs a relatively large surface area so that it can push a reasonable amount of air backwards and forwards. The top plate is made so that it can vibrate up and down relatively easily. On the inside of the plate is a series of braces. These strengthen the plate. There are different patterns of bracing a guitar, as can be seen in figure 2. An important function is to keep the plate flat, despite the action of the strings which tends to make the saddle rotate. The braces also affect the way in which the top plate vibrates. The back plate is much less important acoustically for most frequencies, partly because it is held against the player's body. The sides of the guitar do not vibrate much in the direction perpendicular to their surface, and so do not radiate much sound.

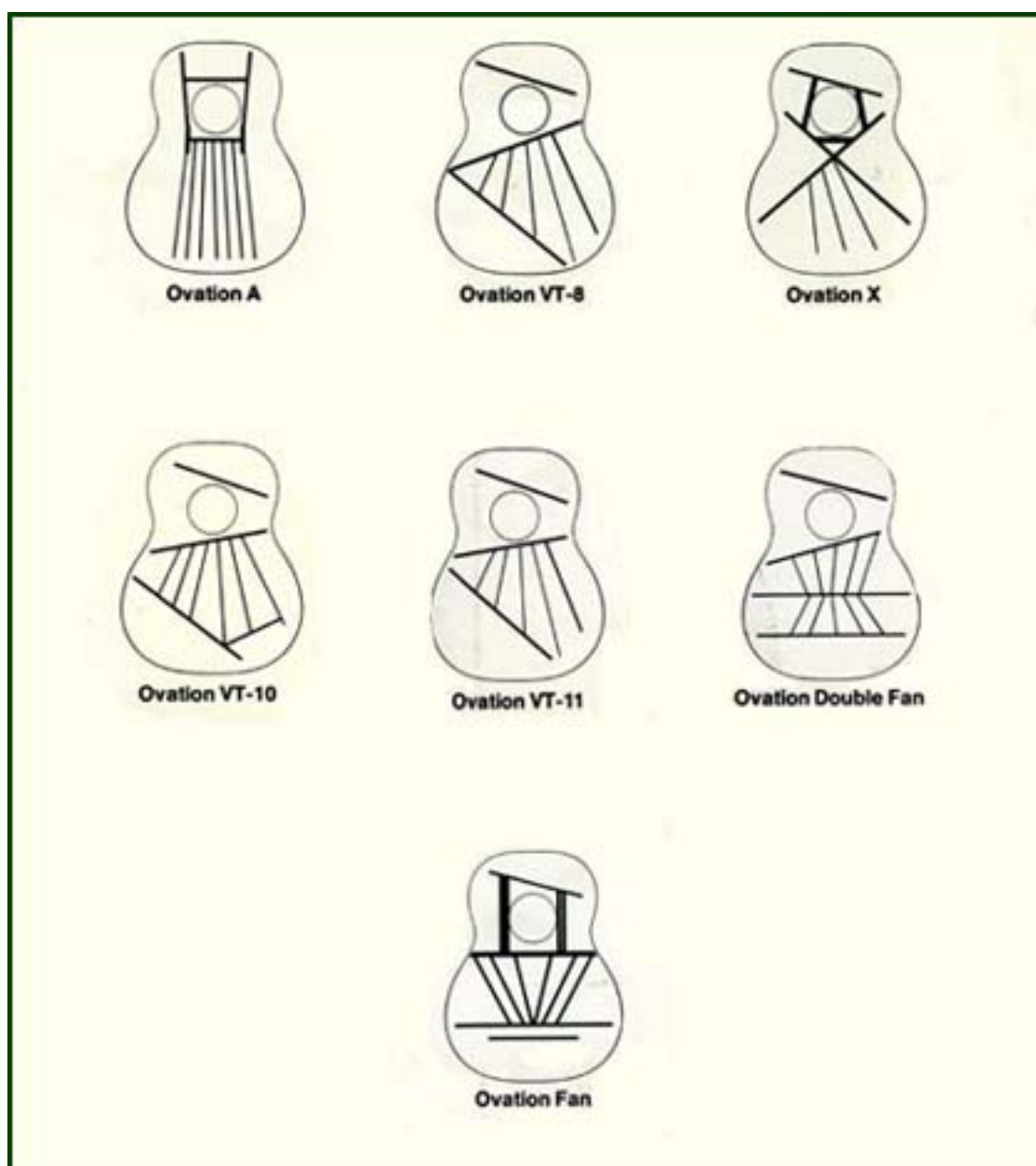


Figure 2: Guitar Bracings

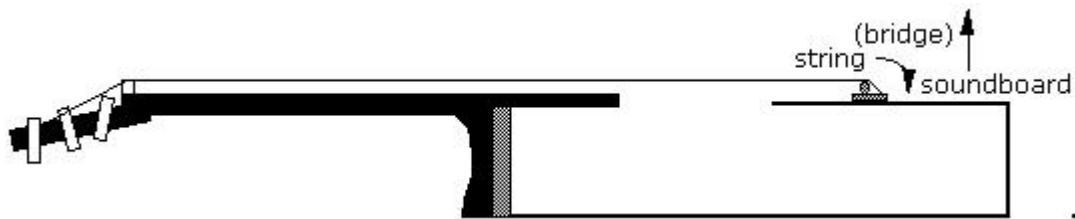


Figure 3: High Frequencies

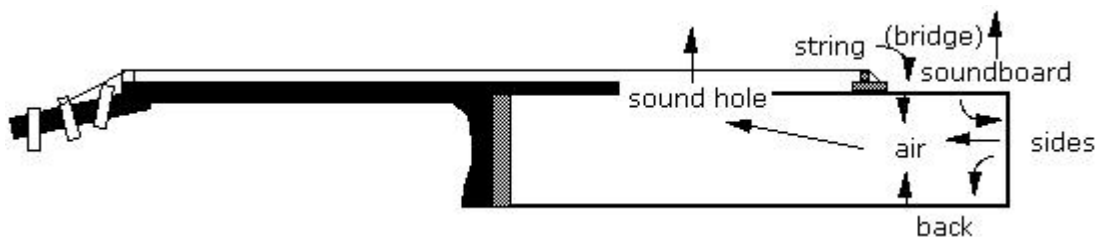


Figure 4: Low Frequencies

3.3 The Guitar As a System of Coupled Vibrators

The guitar can be considered as a system of coupled vibrators. This refers to an interaction between two or more vibrating elements. The plucked strings radiate only a small amount of sound directly, but they excite the bridge and top plate, which in turn transfers energy to the air cavity, ribs and back plate. If these elements interact well, the whole system is said to be *strongly coupled*. The higher frequency sounds are produced by string interaction with the bridge and then the sound board, whereas the lower frequencies are essentially driven by the internal air cavity/sound hole and ribs/back coupling effects, as can be seen in figures 3 and 4.

3.4 The Air Inside and the Helmholtz Resonance

The air inside the body is quite important, especially for the low range on the instrument. It can vibrate a little like the air in a bottle when you blow across the top. In fact if you sing a note somewhere between $F\#2$ and A_2 (it depends on the guitar) while holding your ear close to the sound hole, you will hear the air in the body resonating. The Helmholtz resonance of a guitar is due to the air at the soundhole oscillating, driven by the springiness of the air inside the body. Let's analyse this more in detail, using the concept of blowing air across the top of an open bottle. The air in the body of a guitar works in almost the same way.

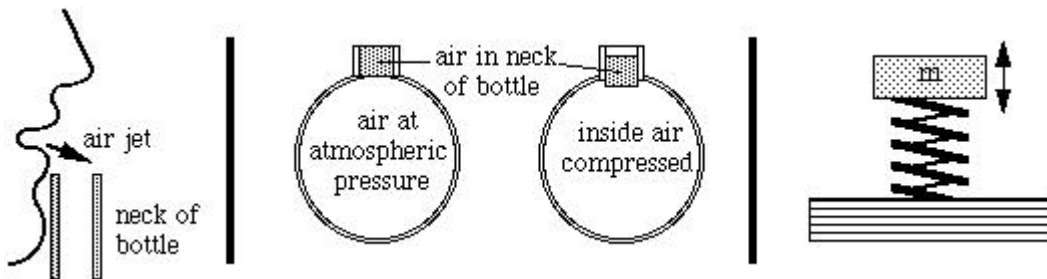


Figure 5: Helmholtz Principle

Let's take a look at figure 5. The vibration here is due to the springiness of air: when you compress it, its pressure increases and it tends to expand back to its original volume. Consider a lump of air at the neck of the bottle. The air jet can force this lump of air a little way down the neck, thereby compressing the air inside. That pressure now drives the lump of air out but, when it gets to its original position, its momentum takes it on outside the body a small distance. This rarifies the air inside the body, which then sucks the lump of air back in. It can thus vibrate like a mass on a spring (at right of figure 5). The jet of air from your lips is capable of deflecting alternately into the bottle and outside, and that provides the power to keep the oscillation going. We analyze this principle quantitatively.

Let the air in the neck have an effective length L and cross sectional area S . Its mass is then (see also figure 6)

$$\begin{aligned} m &= \rho V \\ &= \rho SL \end{aligned}$$

If this plug of air descends a small distance x into the bottle, it compresses the air in the container so that the air that previously occupied volume V now has volume $V - Sx$. Consequently, the pressure of that air rises from atmospheric pressure P_A to a higher value $P_A + p$.

Now we might think that the pressure increase would just be proportional to the volume decrease. That would be the case if the compression happened so slowly that the temperature did not change. In vibrations that give rise to sound, however, the changes are fast and so the temperature rises on compression, giving a larger change in pressure. Technically they are adiabatic, meaning that heat has no time to move, and the resulting equation involves a constant γ , the ratio of specific heats, which is about 1.4 for air (these are physical phenomena we accept; explaining these would drive us too far away from the subject, which is the guitar). As a result, the pressure change p

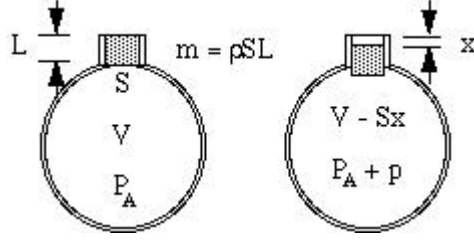


Figure 6:

produced by a small volume change ΔV is just

$$\frac{p}{P_A} = -\gamma \frac{\Delta V}{V} = -\gamma \frac{Sx}{V}.$$

Now we use Newton's law

$$F = ma,$$

or

$$\frac{d^2x}{dt^2} = \frac{F}{m}.$$

Now the pressure generated by the movement of the mass m is given by $p = F/S$. So $F = pS$. Substituting this in Newton's equation yields:

$$\frac{d^2x}{dt^2} = \frac{pS}{\rho SL} = -\frac{\gamma SP_A}{\rho VL}x.$$

So the restoring force is proportional to the displacement. This is the condition for Simple Harmonic Motion, and it has a frequency which is $1/2\pi$ times the square root of the constant of proportionality, so

$$f = \frac{1}{2\pi} \sqrt{\frac{\gamma SP_A}{\rho VL}}.$$

Now the speed c of sound in air is determined by the density, the pressure and ratio of specific heats, so we can write

$$f = \frac{c}{2\pi} \sqrt{\frac{S}{VL}}.$$

Let's look an example: for a 1 litre bottle, with $S = 3$ square centimetres and $L = 5$ centimetres, the frequency is 130 Hz, which is about the note C_3 . So the wavelength is 2.6 metres, which is much bigger than the bottle. This follows the line of expectations, because blowing a bottle generates low sounds. Hence we expected a long wavelength.

4 Vibrating Modes of The Guitar Top Plate

4.1 Introduction to Modes

A guitar top plate vibrates in many modes. The way a guitar is braced has an important influence on the way its top plate vibrates. Because of the way a guitar is played, the back plate is less important.

A mode of vibration is simply a way of vibration. For example, what happens when you strike a xylophone bar in the middle and set it vibrating? The bar is supported at two points towards the ends. The simplest mode of vibration is this: when the middle of the bar goes up (as shown by the solid lines in figure 7) the ends of the bar go down. When the middle goes down (dashed lines), the ends go up. The two points that do not move are called nodes and are marked N in the figure. This first mode of the xylophone bar is rather

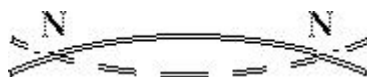


Figure 7: Xylophone Bar Vibrating

similar to a mode of vibration of a simple rectangular plate which is called the (0,2) mode (for a rectangular plate, modes are identified by two numbers (n,m) where n is the number of modes running parallel to the long axis and m the number in the perpendicular direction).

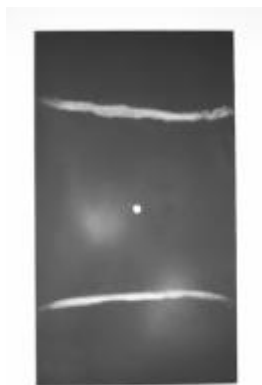


Figure 8: Mode (0,2) of a Uniform Rectangular Aluminium Plate

4.2 Nodes

We already introduced the concepts of nodes. But why are there nodes? The supports of the xylophone bar did not cause the nodes, rather they are placed at the positions which are nodes so as to facilitate this vibration. In an object which is not firmly clamped, a vibration cannot easily move the centre of mass of the object. It follows that, if some part is going up, another part is going down. In the simple motion at resonance, the points that divide these regions are nodes. In simple modes of vibration, the motion of different parts is either exactly in phase or exactly out of phase, and the two regions are separated by nodes.

In figure 8, the lines are formed from sand that has collected at the nodes, but has been shaken off the moving regions. The top plate of a guitar is more complicated in shape, and also depends on the bracing and the presence of a sound hole, so the nodes also have a more complicated shape. An example of this can be seen in figure 9.



Figure 9: Example of a Guitar Top Plate Vibrating Mode

5 Acoustic Concepts

Acoustics refer to the scientific study of sound, especially of its generation, transmission, and reception. The science of acoustics may not be able to answer all of the musical questions asked, but it certainly plays a vital role in objectifying quantities such as frequency, sound intensity, gain, sustain,..., as they are produced by various instruments and techniques of playing. For the guitar specifically, we can examine three acoustical qualities: acoustic radiation patterns, spectral response and intermediate timescale phenomena.

5.1 Acoustic Radiation Patterns

Acoustic radiation patterns are just a map of how the sound intensity varies with the angle and distance from the instrument. They are formed due to the way the guitar vibrates in its various resonant modes. The total sound intensity at your ear is not only dependent upon the frequency the guitar is played at, but at what distance and angle your ear is to the instrument. For example, an A_3 note (110 Hz) may not sound much louder at various angles from the guitar (keeping the same distance), but playing a B_4 (247 Hz), you may find that you hear a noticeably different sound intensity at various angles.

Another example can be found in figure 10. The left diagram shows the sound radiation pattern of 2 sounds with frequencies 102 Hz and 204 Hz. It shows that these sounds have the same intensity, no matter in which angle of the source of the sound you are. So the A_3 note we described above, works in this way. On the contrary, a sound with frequency 376 Hz, as shown in the middle diagram, has an intensity that depends on the angle you are from the source. It has a certain sound radiation pattern that is not a circle. An analogue case in the right diagram, where we observe a sound with a frequency of 436 Hz. The complicated patterns formed in space due to the

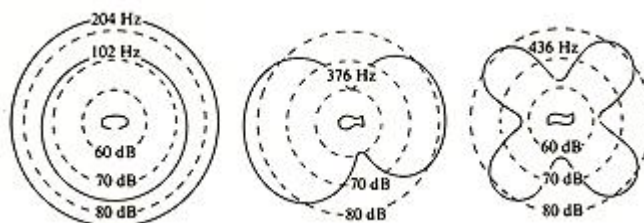


Figure 10: Sound Radiation Patterns

various modes of the guitar is one of the reasons it is so hard to capture that live sound on recordings. A lot of people like listening to live music because of the complex change in sounds as you move around.

5.2 Spectral Response

A sound spectrum is a plot of how the intensity of a particular sound varies with frequency. A useful way of looking at a sound produced by an instrument is to examine its spectral response. A typical way to measure an instrument's spectral response is to excite the instrument with some sort of mechano-acoustic oscillator, such as a speaker attached directly (figure 11).

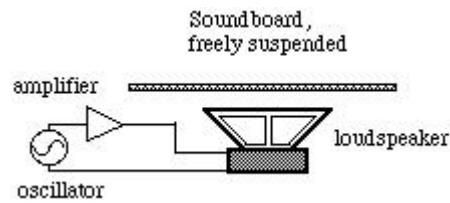


Figure 11: Diagram of Equipment to Measure Spectral Response

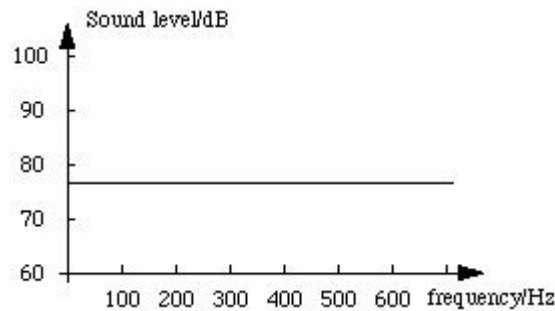


Figure 12: Flat Spectral Response

The oscillator is connected to a function generator (usually connected to a computer) which scans over a frequency range. A transducer such as a microphone converts the resulting output signal into an electrical one, at a fixed point; a spectrum can then be plotted. The spectral response can tell a lot about a particular instrument, showing characteristics, including the main resonances and how sharp they are. A flat response (figure 12) represents an equal intensity of sound produced at every frequency, yet a guitar generally has a series of peaks and valleys (figure 13).

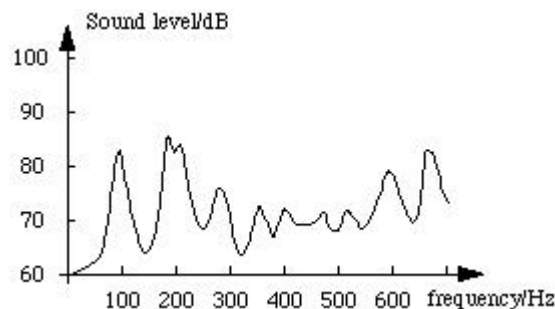


Figure 13: Spectral Response With Peaks and Valleys, Typical For Musical Instruments

5.3 Intermediate Timescale Phenomena

Intermediate timescale phenomena refer to acoustic events noticeable on a more moderate timescale, by which we mean of the order of 10 ms or less. We will mention three of these phenomena: rise time, decay time and envelope effects. Leaving aside the spectral response, the major contribution to timbre is due to these intermediate timescale phenomena.

5.3.1 Rise Time

If we look at a graph of a sound's intensity (loudness), it varies over time (figure 14). The *rise time* is the time it takes for the sound to go from

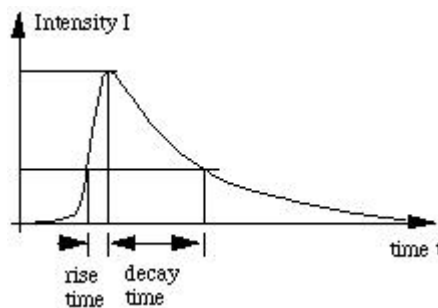


Figure 14: Intensity vs. Time of a Sound Pulse

$1/e$ (= 37%) of the maximum height to the pulse's maximum value. Some noises are perceived as very 'loud' when it has a small rise time (quickly reaches full intensity), yet may have quite a low intensity.

5.3.2 Decay Time

The opposite of the rise time is called the *decay time*. It is often referred to as *sustain*. It is the time needed for the sound to go from its maximum intensity to $1/e$ (= 37%) of this maximum.

Lovers of guitar music tend to prefer a sound that has a relatively long sustain. Guitars that exploit mechanical coupling such as the 12-string produce sounds characterised by a high sustain.

5.3.3 Envelope Effects

The envelope of a wave system indicates how the wave peaks behave. Some types of envelope signatures are quite noticeable in musical sounds. A great example of this is the tremolo effect (figure 15), an effect favoured by 60's

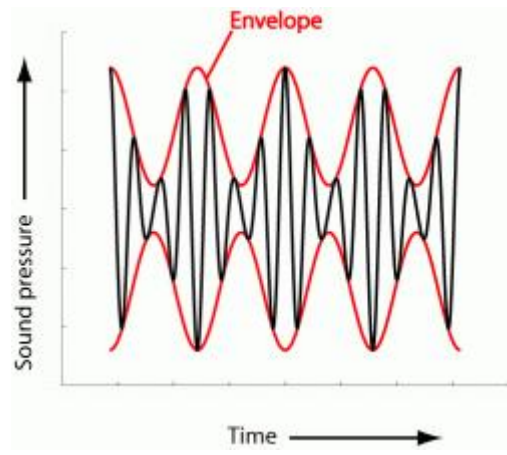


Figure 15: Diagram of the Tremolo Effect

rock guitarists. The sound intensity is smoothly increased and decreased sinusoidally over time, giving a wavy feel to the sound.

6 References

- <http://www.phys.unsw.edu.au/music/>
- Neville H. Fletcher and Thomas D. Rossing, The physics of musical instruments (2nd ed), Springer-Verlag, New York, 1999.